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Discussion paper No. 41

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December 1966

ON GROWTH MODELS AND MULTIPLIERS: THE CASE OF THE MISSING CONSTRAINT

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Paul Clark, Charles Howe, and Hiram Karani have provided us with the key to raising incomes in the East African countries: deficit spending.[1] If, keeping tax rates constant, government current expenditure is raised by Shs.100, income will rise by Shs.166, 160, and 150, respectively, in Tanzania, Uganda, and Kenya. Clearly, if it yields these results, the Clark/Howe/Karani work deserves further study.

To unravel this mystery, let us look at the CHK model. The original Clark version[2] contains 5 exogenous and 34 endogenous variables. Output is disaggregated into 6 sectors, and inter-sector relationships are a feature of the model. Thus if output in (say) the manufacturing sectors is to increase, it requires increases in output in sector supplying manufacturing, as well as increases in imports and the capital stock. One can take any specified level of output and, by working through the inter-relationships specified by the model, find out how this output will be distributed among sectors, and what import and capital stock requirements it implies.

But what is this about multipliers? Clark, Howe, and Karani all examine multipliers: "the ultimate effect of a £1 change in each of the autonomous variables upon GDP, government revenue and

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[1]See Paul Clark, "The Rationale and Uses of a Projection Model for the East African Economies, " EAER, vol. I, N.S., No.2, June 1965; Charles Howe and Hiram Karani, "A Projection Model for the Kenya Economy: A study in Development Planning and Comparative Economic Structures," EAER, vol.1, N.S.,No.2, June 1965. For a study very similar to the Clark/Howe/Karani work, see I.M.A.Penguin, "A Projection Model for Queen Maud Land," Review of Antarctic Political Economy, June 1931.

[2]The Howe-Karani version is an adaption for the Kenya economy of the Clark version; the changes are marginal.

imports."[1] Or alternatively, "these multipliers ... are the parameters of the reduced form of the model ..."[2] Thus Clark calculates the effect of changes in exports and government expenditure on GDP, government revenue, and imports. Howe and Karani, not satisfied with this, calculate the effect of changes in each of 7 variables on each of 13 other variables.

To understand what this implies, let us consider the question of constraints. In a Keynesian-type model, an increase in government expenditure (or in autonomous spending generally) will increase income by some multiple of the initial expenditure increase. But, it is assumed in a Keynesian model that output is not supply constrained; the only constraint on output is spending.

By contrast, in most LDCs, it is believed that output is constrained by supply considerations. Thus Clark says[3] that "the model is designed to emphasize three potential constraints on development expenditures and policies: the balance of trade, which depends mainly on the various import parameters; the government budget surplus or deficit, which depends mainly on the tax revenue parameters; and the required saving, which depends mainly on the capital formation parameters."

Clark introduces these constraints into the model as follows. He writes[4]

$$I \geq \delta \Delta Y \qquad (1)$$

where I = investment and
Y = income.

Then, to eliminate the nonlinearity introduced in (1), this ineguality is approximated by

$$I \geq \delta * Y \qquad (1a)$$

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[1] Clark, page 7.

[2] Howe and Karani, page 26.

[3] Clark, page 2

[4] This is a fairly liberal interpretation of Clark's model, but I believe it does him justice. For ease of exposition, I have aggregated relationships that appear in disaggregated form in the Clark work, but this does not affect the argument.

where δ^* is a function (determined outside the model) of the income growth rate. Inequality (1a) gives the minimum investment requirements.

Import requirements are given by

$$M = \mu Y \tag{2}$$

where M = imports. Now imports and investment are defined:

$$I = S + F - D \tag{3}$$

$$M = X + F \tag{4}$$

where S = saving, F = foreign capital imports, D = government deficit, and X = exports.

Combining (1a) and (3),

$$Y \leq \frac{S + F - D}{\delta^*} \tag{5}$$

And combining (2) and (4)

$$Y < \frac{X + F}{\mu} \tag{6}$$

More precisely, output can be expressed

$$Y = \min \left[\frac{S + F - D}{\delta^*} , \frac{X + F}{\mu} \right] \tag{7}$$

that is, as the minimum of output permitted by the investment and import constraints, respectively. Only one of these constraints need be binding at any point in time, although it is possible that they may be simultaneously binding.[1]

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[1]This model permits a situation in which domestic resources are redundant because of a foreign exchange shortage. One would expect in practice that methods could be found for using the excess domestic resources to produce either exports or import substitutes -- even if at a loss -- and accordingly permit some expansion in output.

The government budget constraint enters in a different way. Government revenue, R is expressed as a function of income,

$$R = \beta Y \tag{8}$$

and the following identity is written

$$R = G - D \tag{9}$$

where G = government (current account) spending. Now, R does not affect the level of output directly, but is likely to affect the economy's capacity to save. Write

$$S = \alpha(1 - \beta) Y \tag{10}$$

Then if, to finance a higher level of government expenditure, β is raised, α must also be raised to maintain saving at its previous level. Thus the government budget constraint is merged with the saving constraint, and can be ignored in the remainder of this note.

Equation (7) expresses output as a function of S, F, D, and X. If values are assigned to these variables, then at least one of the two constraints is effective, and output determined. Alternatively, one can specify a target value of output and determine the required minimum values of S + F - D and X + F. If S is made endogenous via a saving behavioral relationship, such as equation (10), then given the marginal saving propensity, α , one can determine the values of F, X, and D required to obtain target output.

Now an output multiplier is the total derivative of output with respect to an exogenuous variable. The value is necessarily zero if the variable enters only into a nonbinding constraining relationship. For example, if the balance of payments is effectively constraining output, and increase in S will not affect output; but an increase in either X or F will increase output. And the multiplier, if one wishes to use the term, gives the relationship. Thus, in a simple model, with a (total) marginal import propensity of $\frac{1}{2}$ in all sectors, and with an effective BOP constraint, an increase of £1 in exports will increase output by £2. The lower the MPM, the higher the multiplier. Note two points: (1) the multiplier can relate output only to an exogenous variable (one that enters into a constraint); (2) the multiplier is non-zero only when the constraint is effective.

Clarke, Howe, and Karani are interested in finding the implications of achieving a target level of output on the BOP, savings ratio, and government deficit. There is nothing wrong with this, but one should not then turn around and calculate multipliers at the same time. By assumption, in this formulation of the problem, there are no effective supply constraints, so that output must be demand-determined. It is for this reason that an increase in G will increase Y. It will also increase requirements on S and F, but who cares? Clark does note this point in passing but appears not to let it concern him; Howe and Karani do not even note it, and the text suggests they are unaware of it. For example,[1] "Government expenditures themselves lead to an increase in revenues in the amount of 35 per cent of the expenditures - a fact to be taken into account in the budgetary process." But surely we are not getting something for nothing. Rather, an increase in G will require increases in other variables such as F and S. If these are free goods, then output is not constrained, and the multipliers are as CHK suggest. If not free goods, then the multipliers are highly misleading. It is more correct to say that an increase in G will <u>decrease</u> output (negative multiplier) unless there are compensating changes in other variables, such as domestic savings or foreign borrowing.[2]

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[1] Howe and Karani, page 27.

[2] Another disturbing result obtained by Clark, related to the discussion above, is that an increase in real agricultural output at constant prices will have a more beneficial effect on income than an increase in the export price. This apparently results from the fact that transportation requirements are generated by the former but not by the latter; and by the fact that an increase in transportation output will generate demands elsewhere in the economy -- with a consequent increase in the requirements of saving and foreign exchange.